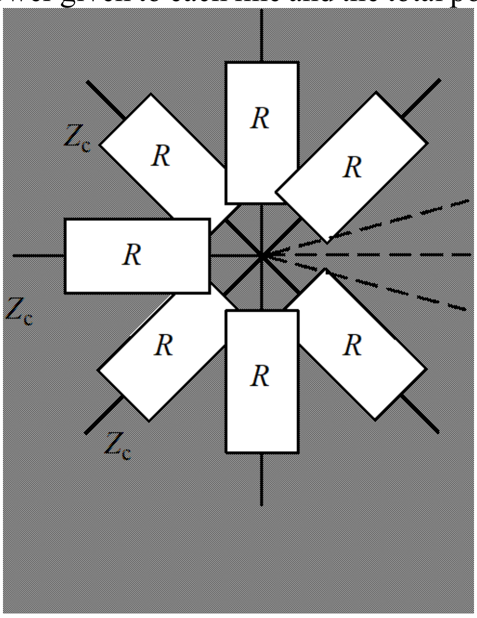
Microwaves

Series 9, solutions

Problem 1

Study the symmetric resistive divider with N ports. Give the value of the N resistors (in series), so that the component is matched at all its ports. Find the scattering matrix of this component, and as a function of N, the power given to each line and the total power absorbed by the divider.



Every input is coupled to N-1 outputs, so that the input impedance, when all the outputs are terminated by matched loads is given by :

$$Z_{in} = R + \frac{1}{(N-1)\frac{1}{R+Z_c}} = R + \frac{R+Z_c}{(N-1)} = \frac{NR+Z_c}{(N-1)} = Z_c$$

In order that the divider to be matched, this input impedance has to be equal to the characteristic impedance, thus :

$$R = \frac{1}{N}\left[(N-1)Z_c - Z_c\right] = \frac{N-2}{N} Z_c$$

The current I injected at the input is equally divided between the N-1 outputs. The power dissipated in the i[th] termination is does given by :

$$P_i = Z_c I_i^2 = Z_c \left(\frac{I}{N-1} \right)^2$$

That must be compared to the total input power, so that

$$\frac{P_i}{P} = \frac{Z_c I_i^2}{Z_c I^2} = \left(\frac{1}{N-1} \right)^2$$

And choosing judiciously the reference planes, the scattering matrix of the divider is given by:

$$\frac{1}{N-1}\begin{pmatrix} 0 & 1 & 1 & \dots & 1 & 1 \\ 1 & 0 & 1 & \dots & 1 & 1 \\ 1 & 1 & 0 & \dots & 1 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & 1 & \dots & 0 & 1 \\ 1 & 1 & 1 & \dots & 1 & 0 \end{pmatrix}$$

The relative power dissipated in all the loads is thus :

$$(N-1)\left(\frac{1}{N-1} \right)^2 = \frac{1}{N-1}$$

So that the relative power dissipated in the power divider is given by :

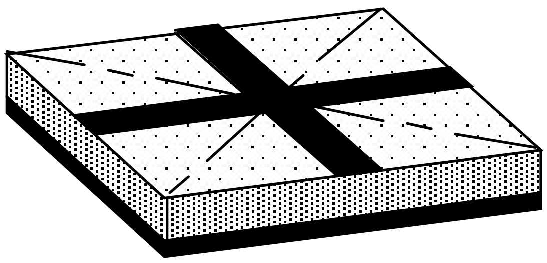
$$1 - \frac{1}{N-1} = \frac{N-1-1}{N-1} = \frac{N-2}{N-1}$$

The results for several values of N are given in the following table :

N	3	4	5	6	7
R	$0{,}333 \cdot Z_c$	$0{,}5 \cdot Z_c$	$0{,}6 \cdot Z_c$	$0{,}666 \cdot Z_c$	$0{,}714 \cdot Z_c$
P_i/P	0,25	0,1111	0,0625	0,04	0,02777
P_d/P	0,5	0,6666	0,75	0,8	0,83333

Problem 2

Determine the scattering matrix of the four port made of the crossing of two 50Ω microstrip lines. The lines are supposed lossless, and the impedance at the reference planes is 50 Ohms.



Let us consider one line as the input. When the three output lines are terminated by a matched load, the impedance of these three output lines seen by the input line is given by

$$50/3 = 16{,}666\ \Omega$$

Which, incidentally, corresponds to a VSWR of 3, and to a reflection coefficient of

$(16{,}666 - 50)/(16{,}666 + 50) = -1/2 = \underline{S}_{ii}$.

due to the symmetry, the transmitted power is equally divided between the three outputs, and we obtain

$$|\underline{S}_{ij}|^2 = (1 - |\underline{S}_{ii}|^2)/3 = 1/4$$

and thus $\underline{S}_{ij} = 0{,}5$. We have supposed in this solution that the reference planes are located $n\lambda/2$ away from the junction, leading thus to real values of the reflection and transmission coefficients.

This problem could also be solved using the double axial symmetry. The symmetry axes are shown in dashed lines on the figure. We find then, replacing the symmetry axes by open or short circuit planes, that

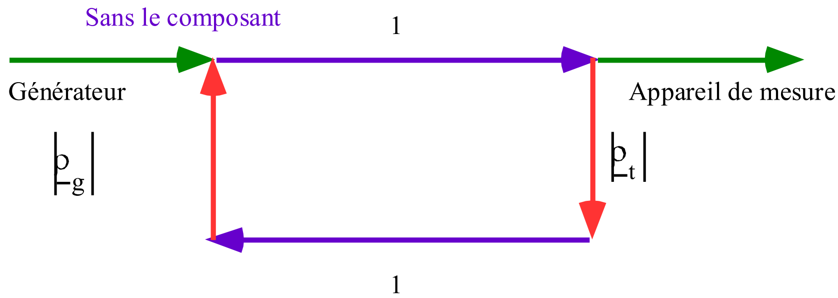
$\underline{\rho}_{ss} = 1$ and $\underline{\rho}_{as} = \underline{\rho}_{sa} = \underline{\rho}_{aa} = -1$,

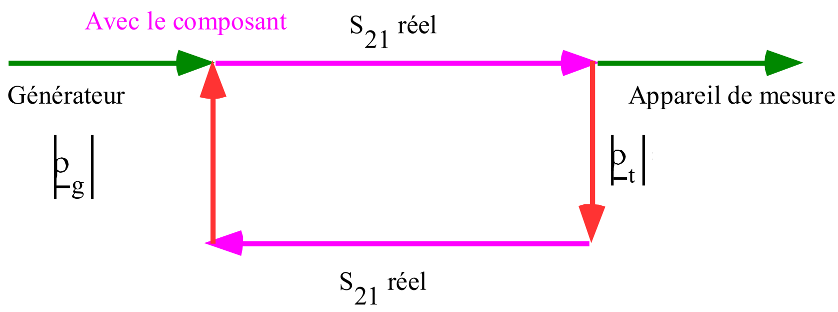
which leads to $\underline{S}_2 = \underline{S}_3 = \underline{S}_4 = 1/2$ and $\underline{S}_1 = -1/2$.

Problem 3

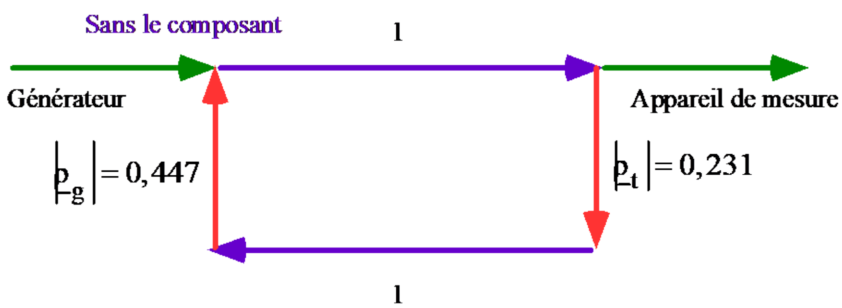
A reciprocal two port device absorbs the quarter of the incident power and is matched at its two ports. We measure its attenuation using an unmatched detector, which VSWR is equal to 1.6. We know also that the generator used for the measurement reflects 20% of the power which is returned to it. We do however not know the phase shift produced by the two port, nor the phases of the reflection at the detector or the generator. Find between which values the *measured* attenuation will lie.

Hint : use the two following flow charts

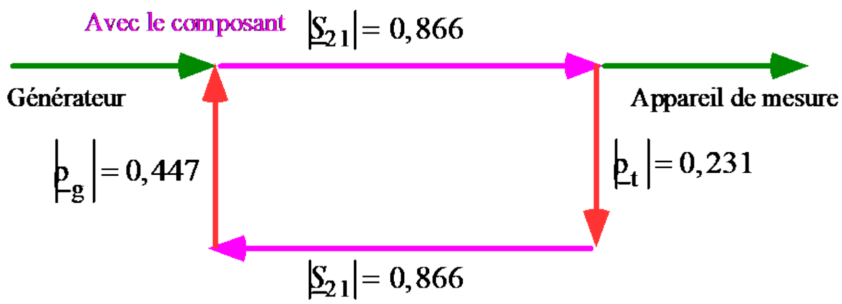




The attenuation of matched two port is given by the division of the power flowing into the load in the absence of of the two port by the power reaching the load in the presence of the two port (note: this works only when the two port is matched)
The reflection coefficient of the detector is $|\rho_t| = 0{,}6/2{,}6 = 0{,}231$, and the one of the generator is $|\rho_g| = \sqrt{0{,}2} = 0{,}447$. The power detected in the absence of the device under test, normalized with respect to the power incident on a matched load lies between the following limits :
$\left(1-|\rho_t|^2\right)\Big/\left(1\pm|\rho_g||\rho_t|\right)^2$, Thus between 0,777 and 1,177.



With the component to measure, which real value of S_{21} is given by $|S_{21}| = \sqrt{0{,}75} = 0{,}866$, these limits become
$0{,}75\left(1-|\rho_t|^2\right)\Big/\left(1\pm 0{,}75|\rho_g||\rho_t|\right)^2$, they lie thus between 0.6116 and 0,8342.



The smallest power ratio as defined above is given by 0.6116/1.177 = 0,5196, which corresponds to a measured $|S_{21}|$ = 0,721 and an attenuation of 2,84 dB.

The largest power ratio is equal to 0,8342/0,777 = 1.074, which corresponds to $|S_{21}|$ = 1.0362 **and to a gain of 0.3 dB** !!!

In the absence of reflection at the load and the charge, we obtain the real value of the power ration, which is 0,75, yielding $|S_{21}|$ = 0,866 and the attenuation is of 1,25 dB. We have thus an error margin of + 1,59/–1,28 dB. The error is as large as the attenuation we want to measure, which shows the importance to match the measurement instrumentation and the generators.